QUANTITATIVE MODULE
Quantitative Decision-Making Aids

In this module, we’ll look at several decision-making aids and techniques, as well as some popular tools for managing projects. Specifically we’ll introduce you to payoff matrices, decision trees, break-even analysis, ratio analysis, linear programming, queuing theory, and economic order quantity. The purpose of each of these methods is to provide managers with a tool to assist in the decision-making process and to provide more complete information to make better-informed decisions.

Payoff Matrices

In Chapter 4, we introduced you to the topic of uncertainty and how it can affect decision-making. Although uncertainty plays a critical role by limiting the amount of information available to managers, another factor is the manager’s psychological orientation. For instance, the optimistic manager will typically follow a maximax choice (maximizing the maximum possible payoff); the pessimist will often pursue a maximin choice (maximizing the minimum possible payoff), and the manager who desires to minimize his “regret” will opt for a minimax choice. Let’s briefly look at these different approaches using an example.

Consider the case of a marketing manager at the Mastercard division of HSBC in Vancouver. He has determined four possible strategies (we’ll label these S1, S2, S3, and S4) for promoting the bank’s Mastercard card throughout Western Canada. However, he is also aware that one of his major competitors, Visa, has three competitive strategies of its own (CA1, CA2, and CA3) for promoting its own card in the same region. In this case, we’ll assume that the Mastercard manager has no previous knowledge that would allow him to place probabilities on the success of any of his four strategies. With these facts, the Mastercard manager formulates the
matrix in Exhibit QM-1 to show the various Mastercard strategies and the resulting profit to the bank depending on the competitive action chosen by Visa.

In this example, if our Mastercard manager is an optimist, he’ll choose S4 because that could produce the largest possible gain: ($28 million). Note that this choice maximizes the maximum possible gain (maximax choice). If our manager is a pessimist, he’ll assume only the worst can occur. The worst outcome for each strategy is as follows: S1 = $11 million; S2 = $9 million; S3 = $15 million; and S4 = $14 million. These are the most pessimistic outcomes from each strategy. Following the maximin choice, the manager would maximize the minimum payoff—in other words, he’d select S3.

In the third approach, managers recognize that once a decision is made it will not necessarily result in the most profitable payoff. There may be a “regret” of profits forgone (given up)—regret referring to the amount of money that could have been made had a different strategy been used. Managers calculate regret by subtracting all possible payoffs in each category from the maximum possible payoff for each given—in this case, for each competitive action. For our Mastercard manager, the highest payoff, given that Visa engages in CA1, CA2, or CA3, is $24 million, $21 million, or $28 million, respectively (the highest number in each column). Subtracting the payoffs in Exhibit QM-1 from these figures produces the results in Exhibit QM-2. The maximum regrets are S1 = $17 million; S2 = $15 million; S3 = $13 million; and S4 = $7 million. The minimax choice minimizes the maximum regret, so our Mastercard manager would choose S4. By making this choice, he’ll never have a regret of profits forgone of more than $7 million. This result contrasts, for example, with a regret of $15 million had he chosen S2 and Visa had taken CA1.

**Decision Trees**

Decision trees are a useful way to analyze hiring, marketing, investment, equipment purchases, pricing, and similar decisions that involve a progression of decisions. They’re called decision trees because, when diagrammed, they look a lot like a tree with branches. Typical decision trees encompass expected value analysis by assigning probabilities to each possible outcome and calculating payoffs for each decision path.

Exhibit QM-3 illustrates a decision facing Mike Rosenthal, the Atlantic region site-selection supervisor for Chapters bookstore. Mike supervises a small group of specialists who analyze potential locations and make store site recommendations to the Atlantic region’s director. The lease on the company’s store in Bayer’s Road Industrial

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**EXHIBIT QM-1**

**Payoff Matrix for Mastercard**

<table>
<thead>
<tr>
<th>Mastercard Marketing Strategy</th>
<th>Visa’s Response (in millions of $)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CA1</td>
</tr>
<tr>
<td>S1</td>
<td>13</td>
</tr>
<tr>
<td>S2</td>
<td>9</td>
</tr>
<tr>
<td>S3</td>
<td>24</td>
</tr>
<tr>
<td>S4</td>
<td>18</td>
</tr>
</tbody>
</table>
Mike’s group has identified an excellent site in a nearby shopping mall closer to downtown. The mall owner has offered him two comparable locations: one with 12,000 square feet (the same as he has now) and the other a larger, 20,000-square-foot space. Mike has an initial decision to make about whether to recommend renting the larger or smaller location. If he chooses the larger space and the economy is strong, he estimates the store will make a $325,000 profit. However, if the economy is poor, the high operating costs of the larger store will mean that the profit will be only $50,000. With the smaller store, he estimates the profit at $240,000 with a good economy and $130,000 with a poor one.

As you can see from Exhibit QM-3, the expected value for the larger store is $239,000 \( (.70 \times 320) \times (.30 \times 50) \). The expected value for the smaller store is $207,000 \( (.70 \times 240) \times (.30 \times 130) \). Given these projections, Mike is planning to recommend the rental of the larger store space. What if Mike wants to consider the implications of initially renting the smaller space and then expanding if the economy picks up? He can extend the decision tree to include this second decision point. He has calculated three options: no expansion, adding 4,000 square feet, and adding 8,000 square feet. Following the approach used for Decision Point 1, he could calculate the profit potential by extending the branches on the tree and calculating expected values for the various options.

**EXHIBIT QM-2**

Regret Matrix for Mastercard

<table>
<thead>
<tr>
<th>Mastercard Marketing Strategy</th>
<th>Visa’s Response (in millions of $)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CA1</td>
</tr>
<tr>
<td>S1</td>
<td>11</td>
</tr>
<tr>
<td>S2</td>
<td>15</td>
</tr>
<tr>
<td>S3</td>
<td>0</td>
</tr>
<tr>
<td>S4</td>
<td>6</td>
</tr>
</tbody>
</table>

Park in Halifax is expiring, and the landlord has decided not to renew it. Mike and his group have to make a relocation recommendation to the regional director.

As you can see from Exhibit QM-3, the expected value for the larger store is $239,000 \( (.70 \times 320) \times (.30 \times 50) \). The expected value for the smaller store is $207,000 \( (.70 \times 240) \times (.30 \times 130) \). Given these projections, Mike is planning to recommend the rental of the larger store space. What if Mike wants to consider the implications of initially renting the smaller space and then expanding if the economy picks up? He can extend the decision tree to include this second decision point. He has calculated three options: no expansion, adding 4,000 square feet, and adding 8,000 square feet. Following the approach used for Decision Point 1, he could calculate the profit potential by extending the branches on the tree and calculating expected values for the various options.

**EXHIBIT QM-3**

Decision Tree and Expected Values for Renting a Large or Small Space

Break-Even Analysis

How many units of a product must an organization sell in order to break even—that is, to have neither profit nor loss? A manager might want to know the minimum number of units that must be sold to achieve his or her profit objective or whether a current product should continue to be sold or should be dropped from the organization’s product line. **Break-even analysis** is a widely used technique for helping managers make profit projections.²

Break-even analysis is a simplistic formulation, yet it is valuable to managers because it points out the relationship among revenues, costs, and profits. To compute the break-even point (BE), the manager needs to know the unit price of the product being sold (P), the variable cost per unit (VC), and the total fixed costs (TFC).

An organization breaks even when its total revenue is just enough to equal its total costs. But total cost has two parts: a fixed component and a variable component. Fixed costs are expenses that do not change, regardless of volume, such as insurance premiums and property taxes. Fixed costs, of course, are fixed only in the short term because, in the long run, commitments terminate and are thus subject to variation. Variable costs change in proportion to output and include raw materials, labour costs, and energy costs.

The break-even point can be computed graphically or by using the following formula:

\[
BE = \frac{TFC}{P - VC}
\]

This formula tells us that (1) total revenue will equal total cost when we sell enough units at a price that covers all variable unit costs, and (2) the difference between price and variable costs, when multiplied by the number of units sold, equals the fixed costs.

When is break-even useful? To demonstrate, assume that, at Todd’s Calgary Espresso, Todd charges $1.75 for each espresso. If his fixed costs (salary, insurance, etc.) are $47,000 a year and the variable costs for each cup of espresso are $0.40, Todd can compute his break-even point as follows: $47,000 / (1.75 – 0.40) = 34,815 (about 670 cups of espresso sold each week), or when annual revenues are approximately $60,926. This same relationship is shown graphically in Exhibit QM-4.

How can break-even serve as a planning and decision-making tool? As a planning tool, break-even analysis could help Todd set his sales objective. For example, he could establish the profit he wants and then work backward to determine what sales level is needed to reach that profit. As a decision-making tool, break-even analysis could also tell Todd how much volume has to increase in order to break even if he is currently operating at a loss, or how much volume he can afford to lose and still break even if he is currently operating profitably. In some cases, such as the management of professional sports franchises, break-even analysis has shown the projected volume of ticket sales required to cover all costs to be so unrealistically high that management’s best choice is to sell or close the business.

Ratio Analysis

We know that investors and stock analysts make regular use of an organization’s financial documents to assess its worth. These documents can be analyzed by managers as planning and decision-making aids.

**break-even analysis**
A technique for identifying the point at which total revenue is just sufficient to cover total costs.
Managers often want to examine their organization’s balance and income statements to analyze key ratios, that is, to compare two significant figures from the financial statements and express them as a percentage or ratio. This practice allows managers to compare current financial performance with that of previous periods and other organizations in the same industry. Some of the more useful ratios evaluate liquidity, leverage, operations, and profitability. These are summarized in Exhibit QM-5.

What are liquidity ratios? Liquidity is a measure of an organization’s ability to convert assets into cash so that debts can be met. The most popular liquidity ratios are the current ratio and the acid test ratio.

The current ratio is defined as the organization’s current assets divided by its current liabilities. Although there is no magic number that is considered safe, the accountant’s rule of thumb for the current ratio is 2:1. A significantly higher ratio usually suggests that management is not getting the best return on its assets. A ratio at or below 1:1 indicates potential difficulty in meeting short-term obligations (accounts payable, interest payments, salaries, taxes, and so forth).

The acid-test ratio is the same as the current ratio except that current assets are reduced by the dollar value of inventory held. When inventories turn slowly or are difficult to sell, the acid-test ratio may more accurately represent the organization’s true liquidity. That is, a high current ratio that is heavily based on an inventory that is difficult to sell overstates the organization’s true liquidity. Accordingly, accountants typically consider an acid-test ratio of 1:1 to be reasonable.

Leverage ratios refer to the use of borrowed funds to operate and expand an organization. The advantage of leverage occurs when funds can be used to earn a rate of return well above the cost of those funds. For instance, if management can borrow money at 6% and can earn 10% on it internally, it makes good sense to borrow—but there are risks to over-leveraging. The interest on the debt can be a drain on the orga-
nization’s cash resources and can, in extreme cases, drive an organization into bank-
ruptcy. The objective, therefore, is to use debt wisely. Leverage ratios such as debt-to-
assets ratio (computed by dividing total debt by total assets) or the times-interest-earned
ratio (computed as profits before interest and taxes divided by total interest charges)
can help managers control debt levels.

Operating ratios describe how efficiently management is using the organization’s
resources. The most popular operating ratios are inventory turnover and total assets
turnover.

The inventory turnover ratio is defined as revenue divided by inventory. The
higher the ratio, the more efficiently inventory assets are being used. Revenue divid-
ed by total assets represents an organization’s total assets turnover ratio. It measures
the level of assets needed to generate the organization’s revenue. The fewer assets
used to achieve a given level of revenue, the more efficiently management is using the
organization’s total assets.

Profit-making organizations want to measure their effectiveness and efficiency.
Profitability ratios serve such a purpose. The better known of these are profit-margin-
on-revenues and return-on-investment ratios.

Managers of organizations that have a variety of products want to put their efforts
into those products that are most profitable. The profit-margin-on-revenues ratio,
computed as net profit after taxes divided by total revenues, is a measure of profits per dollar revenues.

One of the most widely used measures of a business firm’s profitability is the return-on-investment ratio. It’s calculated by multiplying revenues/investments by profits/revenues. This percentage recognizes that absolute profits must be placed in the context of assets required to generate those profits.

**Linear Programming**

Natalie Lopez owns a software development company. One product line involves designing and producing software that detects and removes viruses. The software comes in two formats: Windows and Mac versions. She can sell all of the products she can produce. That, however, is her dilemma. The two formats go through the same production departments. How many of each type should she make to maximize her profits?

A close look at Lopez’s operation tells us she can use a mathematical technique called *linear programming* to solve her resource allocation dilemma. As we will show, *linear programming* is applicable to her problem, but it cannot be applied to all resource allocation situations. Besides requiring limited resources and the objective of optimization, it requires that there be alternative ways of combining resources to produce a number of output mixes. There must also be a linear relationship between variables. This means that a change in one variable will be accompanied by an exactly proportional change in the other. For Lopez’s business, this condition would be met if it took exactly twice the time to produce two diskettes—regardless of format—as it took to produce one.

Many different types of problems can be solved with linear programming. Selecting transportation routes that minimize shipping costs, allocating a limited advertising budget among various product brands, making the optimum assignment of personnel among projects, and determining how much of each product to make with a limited number of resources are just a few. To give you some idea of how linear programming is useful, let’s return to Lopez’s problem. Fortunately, the problem is relatively simple, so we can solve it rather quickly. For complex linear programming problems, computer software has been designed specifically to help develop solutions.

First, we need to establish some facts about Lopez’s business. She has computed the profit margins to be $18 for the Windows format and $24 for the Mac. She can therefore express her objective function as maximum profit = $18 R + $24 S, where R is the number of Windows diskettes produced and S is the number of Mac diskettes. In addition, she knows how long it takes to produce each format and the monthly production capacity for virus software [2400 hours in design and 900 hours in production] (see Exhibit QM-6). The production capacity numbers act as constraints on her overall capacity. Now Lopez can establish her constraint equations:

\[
4R + 6S \leq 2,400 \\
2R + 2S \leq 900 
\]

Of course, because a software format cannot be produced in a volume less than zero, Lopez can also state that R \(\leq 0\) and S \(\leq 0\). She has graphed her solution as shown in Exhibit QM-7. The shaded area represents the options that do not exceed the capacity of either department. What does this mean? We know that total design capacity is 2400 hours. So if Lopez decides to design
only DOS format, the maximum number she can produce is 600 (2400 hours × 4 hours of design for each Windows version).

If she decides to produce all Mac versions, the maximum she can produce is 400 (2400 hours × 6 hours of design for Mac).

This design constraint is shown in Exhibit QM-7 as line BC. The other constraint Lopez faces is that of production. The maximum of either format she can produce is 450, because each disk takes two hours to copy, verify, and package. This production constraint is shown in the exhibit as line DE. Natalie’s optimal resource allocation will be defined at one of the corners of this feasibility region (area ACFD). Point F provides the maximum profits within the constraints stated. At point A, profits would be zero because neither virus software version is being produced. At points C and D, profits would be $9600 (400 units @ $24) and $8100 (450 units @ $18), respectively. At point F profits would be $9900 (150 DOS units @ $18 + 300 Mac units @ $24).
Queuing Theory

You are a supervisor for a branch of Bank of Montreal outside of Halifax. One of the decisions you have to make is how many of the nine cashier stations to keep open at any given time. Queuing theory, or what is frequently referred to as waiting-line theory, could help you decide.

A decision that involves balancing the cost of having a waiting line against the cost of service to maintain that line can be made easier with queuing theory. This includes such common situations as determining how many gas pumps are needed at gas stations, tellers at bank windows, toll takers at toll booths, or check-in lines at airline ticket counters. In each situation, management wants to minimize cost by having as few stations open as possible yet not so few as to test the patience of customers. In our teller example, on certain days (such as the first of every month and Fridays) you could open all nine windows and keep waiting time to a minimum, or you could open only one, minimize staffing costs, and risk a riot.

The mathematics underlying queuing theory are beyond the scope of this book, but you can see how the theory works in our simple example. You have nine tellers working for you, but you want to know whether you can get by with only one window open during an average morning. You consider 12 minutes to be the longest you would expect any customer to wait patiently in line. If it takes 4 minutes, on average, to serve each customer, the line should not be permitted to get longer than 3 people deep (12 minutes \( \times \) 4 minutes per customer = 3 customers).

If you know from past experience that, during the morning, people arrive at the average rate of 2 per minute, you can calculate the probability \( (P) \) that the line will become longer than any number \( (n) \) of customers as follows:

\[
P_n = \frac{\text{Arrival Rate}}{1 - \text{Service Rate}} \times \left( \frac{n}{\text{Service Rate}} \right)
\]

where \( n = 3 \) customers, arrival rate = 2 per minute, and service rate = 4 minutes per customer. Putting these numbers into the above formula generates the following:

\[
P_n = \frac{2}{1 - 4/4} \times \frac{3}{4/4} = (1/2) \times (8/64) = (8/128) = 0.0625
\]

What does a \( P \) of 0.0625 mean? It tells you that the likelihood of having more than three customers in line during the average morning is 1 chance in 16. Are you willing to live with four or more customers in line 6% of the time? If so, keeping one teller window open will be enough. If not, you will have to assign more tellers to staff them.

Economic Order Quantity Model

One of the best-known techniques for mathematically deriving the optimum quantity for a purchase order is the economic order quantity (EOQ) model (see Exhibit QM-8). The EOQ model seeks to balance four costs involved in ordering and carrying inventory: the purchase costs (purchase price plus delivery charges less discounts); the ordering costs (paperwork, follow-up, inspection when the item arrives, and other processing costs); carrying costs (money tied up in inventory, storage, insurance, taxes, and so forth); and stockout costs (profits forgone from orders lost, the cost of re-establishing goodwill, and additional expenses incurred to expedite late shipments). When these four costs are known, the model identifies the optimal order size for each purchase.
The objective of the economic order quantity (EOQ) model is to minimize the total costs associated with the carrying and ordering costs. As the amount ordered gets larger, average inventory increases and so do carrying costs. For example, if annual demand for an inventory item is 26,000 units, and a firm orders 500 each time, the firm will place 52 \((26,000/500)\) orders per year. This gives the organization an average inventory of 250 \((500/2)\) units. If the order quantity is increased to 2000 units, there will be fewer orders \((13 \ [26,000/2000]\)\) placed. However, average inventory on hand will increase to 1000 \((2000/2)\) units. Thus, as holding costs go up, ordering costs go down, and vice versa. The most economic order quantity is reached at the lowest point on the total cost curve. That’s the point at which ordering costs equal carrying costs—or the economic order quantity (see point Q in Exhibit QM-8).

To compute this optimal order quantity, you need the following data: forecasted demand for the item during the period \((D)\); the cost of placing each order \((OC)\); the value or purchase price of the item \((V)\); and the carrying cost (expressed as a percentage) of maintaining the total inventory \((CC)\). Given these data, the formula for EOQ is as follows:

\[
EOQ = \sqrt{\frac{2 \times D \times OC}{V \times CC}}
\]

Let’s work an example of determining the EOQ. Take Barnes Electronics, a retailer of high-quality sound and video equipment. The owner, Sam Barnes, wishes to determine the company’s economic order quantities of high-quality sound and video equipment. The item in question is a Sony compact radio cassette recorder. Barnes forecasts sales of 4000 units a year. He believes that the cost for the sound system should be $50. Estimated costs of placing an order for these systems are $35 per order and annual insurance, taxes, and other carrying costs at 20% of the recorder’s value. Using the EOQ formula, and the preceding information, he can calculate the EOQ as follows:
The inventory model suggests that it’s most economic to order in quantities or lots of approximately 168 recorders. Stated differently, Barnes should order about 24 (4000/168) times a year. However, what would happen if the supplier offers Barnes a 5% discount on purchases if he buys in minimum quantities of 250 units? Should he now purchase in quantities of 168 or 250? Without the discount, and ordering 168 each time, the annual costs for these recorders would be as follows:

- **Purchase cost:** $50 × $4000 = $200 000
- **Carrying cost:**
  - (average number of inventory units times value of item times percentage)
  - \( \frac{168}{2} \times 50 \times 0.02 = 840 \)
- **Ordering cost:**
  - (number of orders times cost to place order)
  - \( 24 \times 35 = 840 \)
- **Total Cost:**
  - $201 680

With the 5% discount for ordering 250 units, the item cost ($50 x [$50 x 0.05]) would be $47.5. The annual inventory costs would be as follows:

- **Purchase cost:** $47.50 × 4000 = $190 000.00
- **Carrying cost:**
  - \( \frac{250}{2} \times 47.5 \times 0.02 = 1,187.50 \)
- **Ordering cost:**
  - \( 16 (\frac{4000}{250}) \times 35 = 560.00 \)
- **Total cost:**
  - $191 747.50

These calculations suggest to Barnes that he should take advantage of the 5% discount. Even though he now has to stock larger quantities, the annual savings amounts to nearly $10 000. A word of caution needs to be added, however. The EOQ model assumes that demand and lead times are known and constant. If these conditions can’t be met, the model shouldn’t be used. For example, it generally shouldn’t be used for manufactured component inventory because the components are taken out of stock all at once, in lumps or odd lots rather than at a constant rate. Does this mean that the EOQ model is useless when demand is variable? No. The model can still be of some use in demonstrating trade-offs in costs and the need to control lot sizes. However, there are more sophisticated lot-sizing models for handling demand and special situations. The mathematics for EOQ, like the mathematics for queuing theory, go far beyond the scope of this text.

\[
EOQ = \sqrt{\frac{2 \times 4000 \times 35}{50 \times 0.02}}
\]

\[
EOQ = \sqrt{28,000}
\]

\[
EOQ = 167.33 \text{ or } 168 \text{ units}
\]
Notes


2. See, for example, S. Stiansen, “Breaking Even,” *Success*, November 1988, p. 16.


4. We would like to acknowledge and thank Professor Jeff Storm of Virginia Western Community College for his assistance in this example.